

# ANALYSIS OF CLOCK MODELING TECHNIQUES FOR USNO CESIUM MEAN

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## Abstract

*The U.S. Naval Observatory (USNO) maintains an ensemble of commercial cesium frequency standards. The mean timescale generated from this ensemble is used as the long-term frequency reference for all USNO clocks. Maintenance of this mean is currently done in a post-processing fashion with clock models being adjusted as far as 75 days into the past. While this method ensures the resulting mean is stable in the past, it can create errors in the near-real-time determination of the frequency of the mean. This in turn adds to the uncertainty in the determination of the clock frequencies relative to the mean. Multiple methods for adjusting clock models in near real time are examined and tested on actual clock data.*

## INTRODUCTION

The USNO has 60 high performance cesium frequency standards (clocks). These clocks are combined to form a timescale or mean that serves as the long-term frequency reference for USNO. At any given time, approximately 75% of these clocks are weighted in that mean, the remainder unweighted due to hardware failures or behavior deemed uncharacteristic for that clock. One component of the behavior of a clock is the frequency rate model that is used to detrend the clock in the mean computation. If a clock model no longer accurately describes the performance of a clock, then the mean is perturbed. In the current design, automated outlier detection allows the mean computation algorithm to temporarily remove clocks that are diverging from their models, but model adjustments require human intervention. This study is to determine the advantages and disadvantages of moving to an algorithm that would make those adjustments in real-time.

## CURRENT DESIGN

Each of the cesium frequency standards is modeled against the cesium mean as a constant in frequency. All clocks participating in the mean receive the same weight. The weight and clock models are inserted into Percival's algorithm [1] to update the mean. The equation that governs the Percival algorithm is given by

$$z_t = z_{t-\tau} + \sum_{i=1}^N W_i [x_t(i) - x_{t-\tau}(i) + \tau r_i(i)] \quad (1)$$

where  $z$  is reference minus mean,  $W$  is the weight of an individual clock,  $x$  is reference minus clock,  $\tau$  is the time between measurements, and  $r$  is the rate of an individual clock.

Clock models are only modified during a manual recomputation of the mean, and the same holds for

weights. The decision on whether or not to change a clock model is done by comparing the residuals of reference mean minus clock against the clock model. Clock model changes are generally affected between 30 and 60 days into the past. Weight changes are often retroactive as well, although not generally dating as far into the past. By making these changes retroactively, all known errors are removed from the mean, making it as close to truth as possible. Since this mean is used as the reference for characterizations of all clocks [2], the retroactive changes reduce the chance of propagating errors in clock characterizations. Unfortunately, retroactive changes in clock models lead to frequency steps in the past which can create more uncertainty in existing clock models, and the frequency changes create phase steps in the mean at the time of recomputation.

## AUTOMATING PERCIVAL'S ALGORITHM

The first method of automation mimics the current methodology in model determination. Cesiums are characterized against the cesium mean as a constant frequency. In the evaluation with live clock data, each clock is initially assigned the same rate model it has in the current system. At each update of the mean, every clock model is tested. The null hypothesis is that the frequency of the clock is described by the rate model given to that clock. This is tested over various taus (time intervals), and expected clock stabilities over each tau are used to define the significance level needed to reject the null hypothesis. If the null is rejected for a clock at a given interval, the model is re-evaluated over that period against the cesium mean. In the automated mean, the model is not changed in the past. This is due to the potential for frequent steps in phase to the cesium mean.

Two different weighting schemes are also examined. The first scheme weights all clocks participating in the mean equally. This method will be referred to as binary weighting, and this is the weighting scheme employed in the current cesium mean. The second scheme assigns weights in inverse proportion to a clock's Allan variance.

Means are created using data from cesiums over roughly 800 days. Figure 1 compares the Allan deviation of the two automated means and the current cesium mean. It is clear that all are similar in stability. Figure 2 shows the accumulated phase offset of the computed means against the USNO Master Clock corrected for steers done by the Bureau International des Poids et Mesures (BIPM) to align the EAL and TAI with a rate removed. The automated means do not fare as well as the current mean, showing a degree of frequency drift that the current algorithm does not. This frequency drift is in fact introduced by the algorithm since it does not correct known errors in the past.

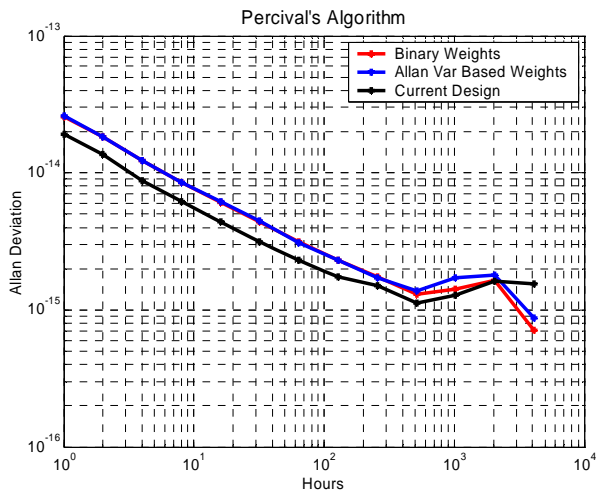


Figure 1

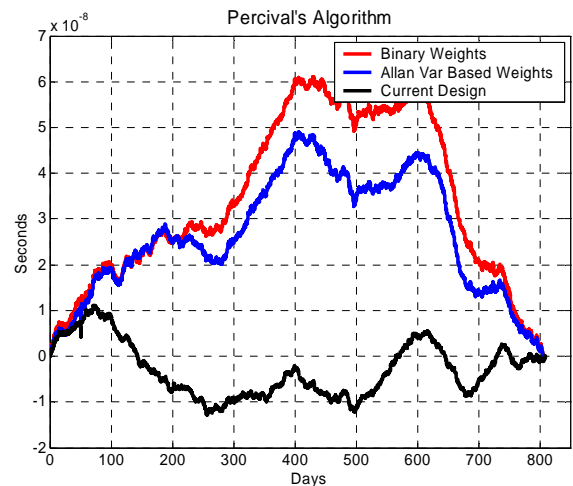


Figure 2

## WEIGHTED MOVING AVERAGE

The second method of automation examined utilizes a weighted moving average to determine the rates for the individual clocks [3]. With each new set of data points collected, the model for each clock is updated by combining the existing rate and the most recent data in a weighted average. Whereas the other methods make the rate changes as discrete steps in frequency, the weighted moving average makes the rates changes as a smooth transition. As in the previous automation, two different weighting schemes are utilized: binary weighting and Allan variance based weighting. The same data sets are used, and the same initialization of clock models is used to determine these.

Figure 3 shows that the stabilities are again similar to that of the current mean, and it also shows that little if anything is gained by varying the weights of clocks within the mean. The accumulated phase plot in Figure 4 shows a frequency drift similar to the one seen in the automation of the Percival algorithm. Again, this frequency drift is introduced by the algorithm.

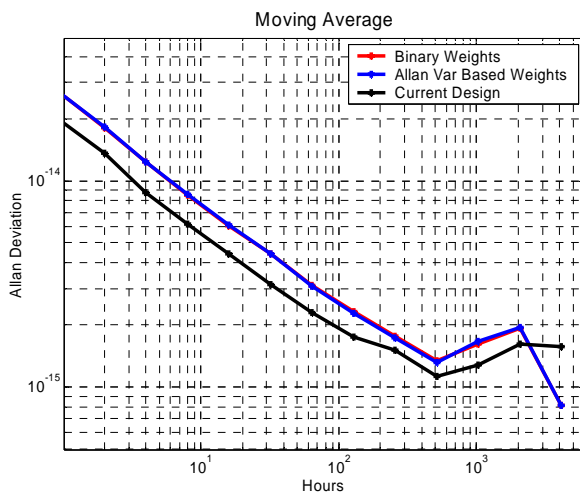


Figure 3

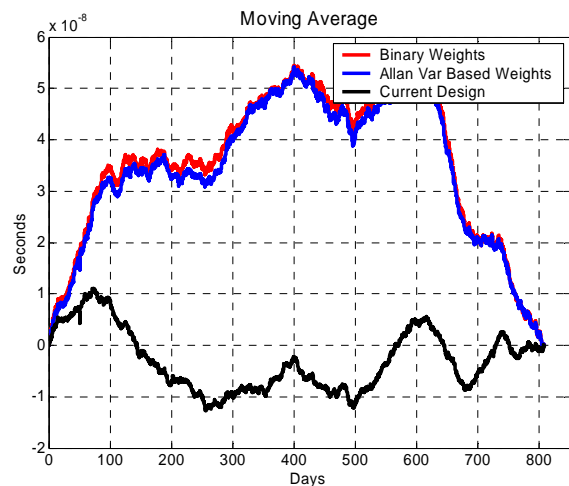


Figure 4

## DELETED MEAN

All of the previous methods use a mean comprised of all weighted clocks as the reference for each individual clock. One concern about this is the correlation between the mean and a clock that is weighted in that mean. A second, related concern is the propagation of error that occurs when a weighed clock is recharacterized against the mean that contains that clock. This brings rise to the idea of a deleted mean. In the current timescale, clocks are removed from the mean used for determining clock models prior to recharacterizing these clocks. By making these adjustments offline, the correlation and propagation of error issues are addressed. It is algorithmically intense to emulate this in real time. A much simpler approach is to consider a deleted mean as a comparator for each weighted clock. In this structure,  $n+1$  means are computed for an  $n$ -clock ensemble. Clock  $i$  is compared to a mean of clocks that excludes clock  $i$ . This is done for each clock, but a mean of the full ensemble is still computed at each step as a reference for clocks not internal to that mean. It is obvious that this idea is not as thorough in its attempt to control the propagation of error as the method employed in the manual recomputations; it does address the bulk of the correlation concern.

Again, the figures compare the performance of each algorithm, now using the idea of the deleted mean, against the current cesium mean. As before, the advantage still goes to the current algorithm, given that it is allowed to make retroactive changes to the mean, where the other algorithms are not. The surprising result from this is that the deleted mean makes essentially no difference to the moving average algorithm. While the introduction of the deleted mean has little effect on the stability of the automated Percival's algorithm, it does make some improvement to the phase offset of the computed means, although there is still a slight drift.

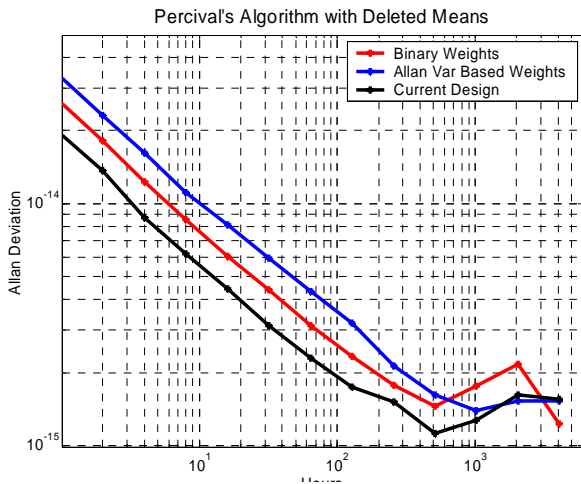


Figure 5

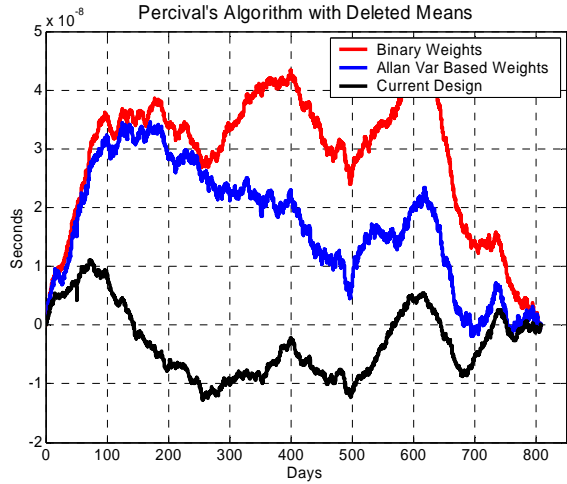


Figure 6

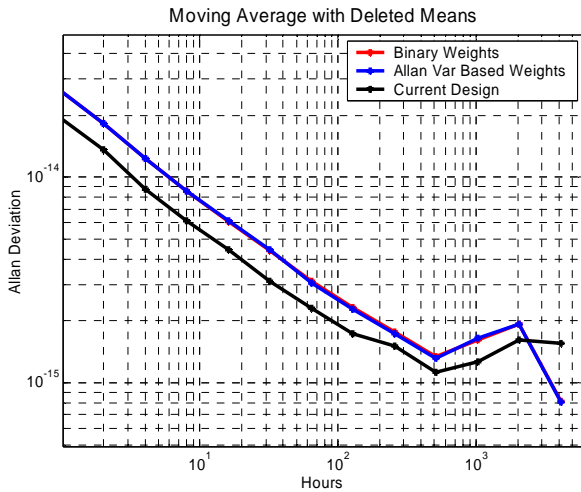


Figure 7

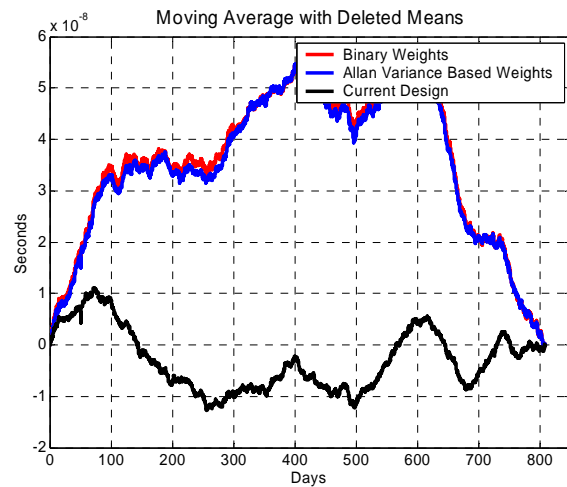


Figure 8

## CONCLUSION

The examined algorithms perform reasonably well from a stability standpoint. The possible solution of running an automated mean alongside the current mean does not look promising at this point due to the phase and frequency offsets that accumulate between the means. Given that the current cesium mean shows no real drift with respect to EAL, it is safe to conclude that the automated algorithms examined are currently inferior to the current mean. That is not to say that the problem has no possible solution. A different algorithm that more closely simulates the manual recomputation may be able to maintain a real-time cesium mean that does not allow internal errors to propagate forward. This result does not adversely affect the current work at USNO on steering a hydrogen maser mean to the cesium mean because of the time constants involved [4].

## REFERENCES

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